Relationships among Pre-service Elementary Education Teachers’ Mathematical Achievement, Science-Based Cognition and Attitude

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Abstract
This study examined 87 elementary pre-service education teachers’ attitudes towards mathematics and science-based cognition – ascertaining the predictive value of these factors on mathematical achievement. In utilizing hierarchical regression analysis, 54% of the variability on mathematical achievement was explained by line graphing, logical thinking and attitude towards mathematics. Attitudes towards mathematics were found to mediate 5-7% of the variability between the line graphing and logical thinking variables with mathematical achievement. Path analysis further substantiated these results, as evidenced by a number of statistically significant goodness-of-fit indices and parameter estimates. As such, 53% of the total variability on the mathematical achievement outcome was attributed to line graphing, logical thinking and attitude towards mathematics. Taken together, results indicate that pre-service teachers having more positive attitudes in mathematics, as well as increased aptitudes for science-based line graphing and logical thinking were more likely to have higher mathematics achievement. Implications for teaching elementary mathematics methods courses are discussed.

Keywords: Pre-service teacher attitudes, mathematical achievement, logical thinking, line graphing.

Introduction

Mathematics and science teacher educators often encourage pre-service teachers to incorporate some type of subject integration into their instruction. Standards for both mathematics and science emphasize the importance of integrating these subjects and recommend the practice (National Council of Teachers of Mathematics [NCTM], 1989, 1995, 2000; National Research Council [NRC], 1989, 1996, 2001; National Science Teachers Association [NSTA], 1992, 1997). Landmark reform documents in each field underscore the importance of blending mathematics and science insofar that one should be dependent on the other, as in the case of the infusion of mathematical knowledge or skill in examining scientific-type data (American Association for the Advancement of Science [AAAS], 1993), as well as incorporating and learning about mathematics by working out problems or applications outside the confines of mathematics (NCTM, 2000). The NCTM (1989) suggests that mathematics education should be rooted in inquiry-based instruction so that students can engage in various forms of investigation, data collection, analysis of results, as well as the communication and visual representation of information. These recommendations stress that educational programs coordinate mathematics and science education as a way to enhance knowledge and understanding in both areas (NRC, 1996). Teachers must possess appropriate levels of content knowledge, pedagogical-content knowledge, pedagogical knowledge (Graham & Fennell, 2001), as well as a repertoire of instructional skills needed to be effective in teaching each respective discipline. These individuals must possess favorable dispositions towards both mathematics (Philipp, 2007) and science (NSTA, 2003).

In order to better understand pre-service teacher content knowledge, attitudes and cognition it is necessary to examine these variables for their corresponding predictive value. By doing so, it
may afford a better understanding of the relationships that are at the crux of mathematical knowledge. In science education, past research has examined various relationships among these factors as they apply to science learning, namely in the context of line graphing and logical thinking (Berg & Phillips, 1994; McKenzie & Padilla, 1984; Wavering, 1989; Yeany, Chin Yap, & Padilla, 1986), as well as with attitudes towards science (Germann, 1998; Oliver & Simpson, 1988; Papanastasiou & Zembylas, 2002). However, intentionally linking such factors to an area other than science has not been evidenced, nor has it taken hold in prediction studies. Therefore, examining science-based line graphing and logical thinking from a mathematics education perspective could offer insights into what influences mathematical knowledge or could be utilized in further defining course teaching ethos.

In developing empirical studies designed to uncover the relationships among pre-service teachers’ knowledge, attitudes and cognition (Adler, Ball, Krainer, Lin, & Novatna, 2005), this study sought to posit that attitudes toward mathematics and cognition (i.e., science-based line graphing and logical thinking) could serve as primary and intermediary influences on elementary pre-service teachers’ mathematics achievement. It is believed that mathematical knowledge is an important feature ingrained within teacher practice. Hence, the purposes of this study are: (1) to investigate the extent of the relationships between elementary pre-service teachers’ subject-matter knowledge in mathematics, attitudes toward mathematics, science-based line graphing and logical thinking abilities; (2) to examine the influences that attitudes towards mathematics, line graphing and logical thinking abilities have on pre-service teachers’ mathematics knowledge, and; (3) to statistically model pre-service teachers’ mathematics achievement as a function of attitudes towards mathematics, line graphing abilities, and logical thinking abilities.

Factors Related to Mathematical Knowledge and Achievement

Mathematical Knowledge

Mathematics teaching is a multifaceted endeavor requiring teachers to possess appropriate levels of content knowledge, pedagogical-content knowledge and pedagogical knowledge (Graham & Fennell, 2001). Consequently, teacher education programs face the challenge of aligning programs in ways to maximize effectiveness in teaching mathematical content, methods or connections to elementary curriculum (Seaman & Szydlik, 2007). Although the acquisition of mathematical knowledge often takes place in highly pedagogical contexts, effective teachers must master basic mathematical ideas in order to make connections to other areas within the field, represent content in different ways, as well as understand the scope and sequence of curriculum and instruction (Ma, 1999). Ball et al. (2005) explain that mathematical knowledge requires, “…fluency, accuracy, and precision in the use of mathematical terms and symbolic notation… [as well as] knowing appropriate representations for a particular mathematical idea…” (p. 4). In developing a model of mathematical learning, Ernest (1989) describes how mathematical knowledge is established early in the life of an individual, and that by the time one reaches the conclusion of pre-service teacher education, it is almost entirely developed. Furthermore, that mathematics teacher knowledge is made up of intricate structures, “characterized by a number of factors, including its extent and depth; its structure and unifying concepts; knowledge of procedures and strategies; links with of other subjects; knowledge about mathematics as a whole and its history” (p. 16).
Regardless of the characterizations, recent evidence indicates that pre-service elementary teachers lack appropriate levels of mathematical content knowledge necessary for successful understanding, learning and subsequent teaching of mathematics as practicing teachers in the general education field (Graham & Fennell, 2001; Koirala & Bowman, 2003; Stinson, Sheats, Harkness, Meyer, & Stallworth, 2009), thus impacting student learning (Hill, Rowan, & Ball, 2005). This suggests that improving mathematical content-knowledge at the pre-service level is a worthwhile pursuit, keeping in mind that multiple forms of conceptualization are required for more complete and accurate portrayals of mathematics. Although past studies have addressed teachers’ knowledge in mathematics as an influential factor on student achievement (Mullens, Murnane, & Willett, 1996; Rowan, Chiange, & Miller, 1997), the explicit link among factors of interdisciplinary influence on pre-service teachers’ mathematical achievement remain undetected and unexplored. In the current study, pre-service teachers’ knowledge is measured in terms of achievement results on a subarea of a basic skills test in mathematics and is believed to be influenced by science-based line graphing and logical thinking abilities.

**Attitude and Achievement in Mathematics**

Studies examining the relationship between attitude and achievement in mathematics have found that attitude can precede achievement (Ma & Kishor, 1997) and that reciprocal relationships (Ma, 1997) and causality (Ma & Xu, 2004) influence achievement outcomes. International evaluation studies indicate positive correlations between attitude and achievement (Kadijevish, 2008; Papanastasiou, 2000), recommending that future research be conducted to uncover additional influences. Arguably, finding sufficient explanations for relationships between these constructs is difficult because of the complexity of their interactions (McLeod 1992; McLeod, 1994). For example, Ma and Kishor (1997) assessed the relationship between attitude toward mathematics and achievement by conducting a meta-analysis of 118 studies. Results indicated that when used as a predictor, attitude towards mathematics was a viable determinant of mathematical achievement when considered alongside problem solving and complex thinking. It was recommended that future studies examine mediating relationships between attitudes towards mathematics and mathematical achievement. Ma and Xu (2004) examined the direction of effects between attitude towards mathematics and achievement in mathematics, finding that achievement preceded attitude as a causal factor for students at the secondary school level. In that case, high achievement preceded positive attitudes. On the other hand, an imbalance of reciprocity between attitude and achievement for elite, high achieving students was also uncovered. Such students enacted high levels of achievement, yet possessed poorer attitudes. Ma and Xu recommended that instructors consider student attitudes during teaching.

These ideas support the notion that attitudes towards mathematics could serve as both direct and mediating variables on mathematics achievement. It is clear that the literature pertaining to the role that attitudes play in the development of proficient mathematics learning is enduring, yet can be categorized as an ill-defined research theme because of its inability to, [be] disentangled from cognition in mathematics education (Niss, 2007, p. 1303). From an interdisciplinary perspective, examining this influence might provide additional insights into explaining an intricate system of knowledge acquisition. This warrants investigation of attitude towards mathematics as a possible factor that could influence a mathematics achievement outcome, considering how closely it appears to be linked with learning.
Line Graphing

One particular area in mathematics that is well exemplified in the standards set forth by the NCTM (1989) is graphing data linearly, which specifies that instructional programs ranging from pre-kindergarten through grade 12 enable all students incorporate this area in order to: (1) represent data using concrete objects, pictures, and graphs (p. 108); (2) represent data using tables and graphs such as line plots, bar graphs, and line graphs; (3) describe the shape and important features of a set of data and compare related data sets, with an emphasis on how the data are distributed; (4) compare different representations of the same data and evaluate how well each representation shows important aspects of the data (p. 177); (5) select, create, and use appropriate graphical representations of data, including histograms, box plots, and scatter plots; and (6) be able to discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stem-and-leaf plots, box plots, and scatter plots (p. 248). Representing these objectives in mathematics teacher education is vital, considering that pre-service teachers will be emphasizing them when teaching about how to visually represent data to future students.

Fry (1984) defines graphs as two-dimensional representations of points, lines and spaces, where data are displayed through represented words and numbers. Characteristically, quantitative data is generated and ultimately transposed into some type of graphical form, where the relationship between the variables becomes visually apparent. This process aids in data interpretation and allows an individual to determine whether there are positive, negative or constant relationships among variables. Specifically, line graphs have been noted as one way to present data between two continuous variables pictorially (McKenzie & Padilla, 1986). According to Friel, Curcio, and Bright (2001), line graphs are the primary form of graph typically found in elementary schools, and learning about them begins as early as kindergarten. Students first learn the fundamentals by plotting points on a line plot diagram in grades K-2. The use of scales in graphing is learned during grades 3-5. By the time students enter grades 6-8, they are expected to organize and present sophisticated and complex data. It is at this point where they are formally introduced to line graphing and are expected to compare data sets. The learning that begins in the earliest grades is essential to the improvement of line graphing skills students are expected to possess as they progress to higher grade levels. Hence, if teaching in this area is to be successful, line graphing must be well understood by general elementary teachers throughout grades K-8.

Logical Thinking

The cognitive mechanisms associated with logical thinking originated with Jean Piaget’s (1964a) theory of cognitive development (TCD), and have been used as the underpinning theory to address conditions needed for learning in both mathematics (Clements, 1984; 2004; Clements & Sarama, 2007; Piaget, 1964b) and science education (Koslowski, 1996; Lawson, 1985; Merz, 1997; Shayer, 2003). Piaget postulates that knowledge development is embedded in action, or operation. These operations are cognitive, and involve actively manipulating pre-existent knowledge in ways that transform thinking and understanding into new conceptual forms. Piaget describes these structures as employed when someone constructs a classification, puts objects in order, counts, measures or reverses actions while adding, subtracting, joining or separating. All of these operations are believed to be linked together and function in non-isolation. The formation, elaboration, organization and functioning of these structures comprise various stages of development unique to every individual and are inherent in logical thinking. As such, an
individual’s cognitive processes progress through four distinctive stages of development (sensory-motor, pre-operational, concrete, and formal or hypothetical-deductive), where each stage is a forerunner to the next. The ability to think logically evolves when that individual experiences active engagement of the mind through mental and physical manipulation of objects. Piaget believed that while constructing knowledge individuals use schemas, or mental structures, to assist in interpreting information. These schemas are conceptions of ideas that one holds in the mind and are critically involved with the process of learning. Studies in science education have utilized Piaget’s TCD in categorizing logical thinking as preoperational reasoning, concrete operational reasoning and formal operational reasoning (Berg & Phillips, 1994), as well as concrete, transitional, or formal operational reasoning (McKenzie & Padilla, 1984). In the current study, logical thinking will reflect these tenets – influential mechanisms inherent of mathematical ability.

**Logical Thinking and Line Graphing in Science**

Several studies in science education highlight the relationships that exist between logical thinking and graphing skills (Berg & Phillips, 1994; Yeany, Chin Yap, & Padilla, 1986; McKenzie & Padilla, 1984; Wavering, 1989). Wavering (1989) found that the level of reasoning necessary to create line graphs was dependent on the student’s grade level, and that as cognition increased, graphing performance increased. McKenzie and Padilla (1984) found that grade 8 students classified as either transitional or formal logical thinkers outperformed those categorized as only concrete operational. Yeany et al. (1986) analyzed different modes of cognitive reasoning and integrated science process skills in establishing a hierarchical structure between the two, finding that each of the science process skills were closely entwined with specific levels of logical thinking. Moreover, that the science process skills involved with graphing require conservational, correlational, and proportional reasoning. Berg and Phillips (1994) investigated the relationship between logical thinking and the ability to construct and interpret line graphs, and determined that logical thinking was an essential component in this process.

The aptitude to think logically in order to substantiate one’s reasoning constitutes mathematical proficiency, and is an important element of mathematical learning (NRC, 2001). This coincides with the type of logical thinking described in science education, where reasoning and formal cognitive operations are required to interpret and solve mathematically based problems in science (Berg & Phillips, 1994; Roadrangka, 1983; Roadrangka, Yeany, & Padilla 1983). These mechanisms of thought are often employed in mathematics during the processing of geometry (Battista, 1990), where logical thinking abilities are resembled as mental problem solving and visualization of objects. Clements and Sarama (2007) describe such thinking as being important because it influences mathematical ability, and that it is closely related to mathematical achievement (Ansari et al., 2003).

Science-based line graphing requires a similar set of mathematically based skill-sets necessary for the processing of representational information. It requires an individual to understand concepts and procedures related to analysis of data and statistics, solving problems using applied mathematical skill (Illinois State Board of Education [ISBE], 2009), visual perception (Friel, Curcio, & Bright, 2001), as well as mental manipulation of variables, data sets, and descriptions of quantified scenarios (McKenzie & Padilla, 1986). Shaugnessy (2007) suggests that graphing is an activity necessary for proper representation of a wide range of data in mathematics, and that understanding various graphs is a vital component of statistical literacy. Similar research in
mathematics education shows that individuals who process information both verbally and logically surpass the performance of those who only employ visual perspicacity (Clements & Battista, 1992).

From a mathematics education standpoint, the integration of scientific-type thinking and reasoning could be demonstrated by emphasizing science-based logical thinking and line graphing. In the current study, logical thinking and line graphing are considered two areas subsumed in the field of mathematics. Hence, logical thinking was viewed as both an intermediary between line graphing and mathematical performance as well as an individual influence. Upon examining the literature pertaining to logical thinking and line graphing, it was clear that no studies specifically addressed the influence that these factors had on the mathematical achievement outcomes of elementary pre-service teachers. It is anticipated that the findings will support the contention that integrating scientific-type thinking and graphical reasoning into mathematics methods teaching could influence mathematics achievement to some extent.

**Theoretical Framework**

Numerous factors may influence a pre-service teacher’s level of mathematical knowledge. Bandura (1986) presented a theory of social cognitive development (SCT) that outlined how cognition, behavior and environmental determinants are integral in shaping how an individual learns. This theory is described in terms of an individual’s ability to control personal thoughts, feelings and actions through a self-guided system. Embodied in this system are affective and cognitive structures, which allow a person to symbolize, use forethought, learn through experience and observation, regulate personal behavior, and engage in self-reflection. The system perceives and evaluates behaviors, which results in the interaction between the self-guided system and the environment in which the person functions. Hence, individuals are given the capability to manipulate both their environment and personal action (Pajares, 1996).

SCT contends that self-referent thought arbitrates between what a person knows and how they act, and that this guides the learning process. In other words, knowledge, skill or past achievement are thought to be incomplete predictors of future performance because the way an individual perceives themselves in their ability can speak more powerfully than other influences or conditions. The constant interactions among cognitive, behavior and environmental factors are therefore part of what Bandura calls reciprocal determinism. When these elements act upon each other, one can assume that a particular outcome can be the result of influences that one or more of these determinants are having. In this study, Bandura’s model of reciprocal determinism was used as a theoretical framework to consider the relationship between pre-service teachers’ mathematical knowledge, attitudes, as well as line graphing and logical thinking abilities.

According to Bandura’s model, a person’s cognitive, or person factors, act as a regulator in determining the probability that an event will occur within the immediate environment. This suggests that the thoughts a person has regarding self-efficacy, beliefs, or attitudes influence the level of performance he or she may experience in a given outcome. As such, individuals base many of their intended behaviors or actions on personal forethought and predictive reasoning. When individuals are in the position to forecast an event or prospect of doing something in the future, they base their judgments in accordance to their level of individual aptitude capable of affecting the environment. If the individual has a high level of aptitude for the activity, they will be more likely to follow through. On the other hand, those who might have a low level of aptitude will be less likely to engage it. Moreover, Bandura (1981) contends that individuals enlist both affective and attitudinal propensities to avoid conditions they believe are beyond their personal capacities, where
they would rather take command of an endeavor they evaluate themselves as attainable. This suggests that individuals not only possess knowledge and skills, but the capability to self-direct and regulate their own course of actions based on other factors.

In the current study, the features of reciprocal determinism translate into how pre-service teachers’ attitudes and cognition influence how they possess and learn mathematics, and subsequently how they achieve. These factors are linked to behavior because the way an individual responds to their perceived experiences or personal abilities are prerequisite to eventual success in practice or purposeful avoidance of teaching mathematics. In other words, having positive or negative attitudes toward mathematics, as well as having higher or lower line graphing or logical thinking abilities are associated with probabilistic behavior outcomes. Attitude and cognition are linked to environmental determinants because the teaching and learning involved with these factors exists in a social milieu, both academic and personal, subjected to the mechanisms of the situation. Overall, investigating these interrelationships is an important step in establishing credence to the notion that certain science-based factors can influence knowledge in an area often regarded as highly interconnected to science.

**Research Questions**

Understanding factors that influence elementary pre-service teachers’ mathematics knowledge is complex and may depend on a both attitude and cognition. However, few studies have attempted to examine or model designated relationships among these variables. This study investigates the following questions:

1. What is the strength of the relationship among attitude towards mathematics, line-graphing ability, logical thinking ability and mathematics achievement?

2. To what extent can mathematics achievement be predicted by attitudes towards mathematics, line graphing ability and logical thinking ability?

3. Considering the mediating roles of attitudes towards mathematics, line graphing and logical thinking, to what extent can direct, indirect and total effects be attributed to these factors as they predict the mathematics achievement outcome?

**Method**

**Sample**

The participants in this study were a sample of convenience taken from the population of pre-service teachers enrolled in two different elementary science methods courses required for a program leading to certification in elementary education at a mid-western research university in the United States. Each of the science education courses included 4 and 2 sections, respectively. The first course emphasized science process skills and inquiry, while the second focused on science content relevant to K-8 teaching. There were 94 initial participants that were predominantly female (81.9%); all were either juniors or seniors. Their ages ranged from 20 to 43 years. Seven female participants were omitted from final data analysis because they were unable to provide the
researcher with a score for the mathematics assessment. Participants were concurrently enrolled in both science and mathematics methods courses at the time of the study.

**Teacher Education Context**

The instructional aspect of the teacher education program takes place over the course of two years, where four sections of required courses in both mathematics and science education are required. Pre-service teacher cohorts partake in a number of classroom observations and student teaching experiences throughout training. During the first semester of enrollment in the program, pre-service teachers carry out 30 hours of individual classroom observations. Upon completion of the program, a total of 40 hours of observation are accumulated. Pre-service teaching experiences are divided between the final two semesters. In the second to final semester, they teach for approximately 35 hours. Subsequently, during the final semester, teaching is entirely in the schools.

**Instruments**

**Attitudes Toward Mathematics Inventory**

Attitudes towards mathematics were measured using the Attitude Towards Mathematics Inventory (ATMI). The ATMI was developed and validated by Tapia and Marsh (2005) for use in assessing pre-service teacher attitudes toward mathematics. A total of 134 college-aged students were used to establish Cronbach alpha reliabilities of .96 for self-confidence, .93 for value, .88 for enjoyment, and .87 for motivation. Correlations for the factors in the model were .52 for self-confidence and value, .75 for self-confidence and enjoyment, .76 for self-confidence and motivation, .63 for value and enjoyment, .65 for value and motivation, and .81 for enjoyment and motivation. Chi-square statistics indicated that the revised instrument was a good fit with the original, (χ² [2, N = 134] = 2.834, p = .242). The ATMI contains 40 items using a 5 point Likert-type scale, ranging from strongly agree to strongly disagree.

**Test of Graphing in Science**

Line graphing ability was measured using the Test of Graphing in Science (TOGS). The TOGS was developed by McKenzie and Padilla (1986). TOGS was developed and validated to measure graphing skills of students ranging from grades 7 through 12 (KR - 20 = .83). TOGS uses twenty-six multiple-choice items to measure objectives related to selecting appropriate axes, locating points on a graph, drawing lines of best fit, interpolating, extrapolating, describing relationships between variables, and inter-relating the data displayed on two graphs. Because it is anticipated that pre-service teachers would be teaching mathematics to students ranging from grades K-8, the complexity of content found in the instrument was determined to be adequate for what would be expected to be taught at those grade levels.

**Group Assessment of Logical Thinking**

Logical thinking ability was measured using the Group Assessment of Logical Thinking (GALT) developed by Roadrangka, Yeany and Padilla (1983). The GALT measures six logical operations based on Jean Piaget’s sensory-motor, pre-operational, concrete, and formal levels of
development (Berg & Phillips, 1994; Yeany et al., 1986). These operations include conservation-reasoning, proportional reasoning, controlling of variables in an experimental situation, probabilistic reasoning, correlational reasoning and combinatorial reasoning. Each of the 12 items in the GALT consists of two multiple-choice items, one for the answer to the item and the other for a reason for the answer. Each multiple-choice item and the accompanying reason are preceded by a pictorial representation of the cognitive task. GALT was administered in a setting where student responses to twenty-one items, as well as their justification for each of the responses, were used to establish construct validity ($\alpha = .71$). Criterion validity was established by administering the GALT alongside the Test of Integrated Process Skills (TIPS II) (Burns, Wise, & Okey, 1983), indicating acceptable total test reliability ($\alpha = .85$).

**Mathematics State Certification Test**

Mathematical content knowledge was measured using a 48-item portion of a basic skills test from the Illinois Certification Testing System (ICTS) (reliability of test: KR-20 = .91). Items reflect content that would most likely be found in grades K-8 according to the content strands advocated by the NCTM (2000) and include number and operations, algebra, geometry, measurement, data analysis, and probability. Specifically, the objectives for the test are as follows: (1) Solve problems involving integers, fractions, decimals, and units of measurement; (2) Apply mathematical reasoning skills to analyze patterns and solve problems; (3) Solve problems involving algebra and geometry; (4) Understand concepts and procedures related to data analysis and statistics; (5) Solve applied problems using a combination of mathematical skills (including word problems involving one and two variables). Test scores for the mathematics subarea were reported on a scale of 100 to 300, with passing scores equal to a scaled score of 240.

**Data Analysis Procedures**

All of the data collection for this study took place during the first month of the academic semester. Data analysis first consisted of correlational analysis in order to uncover which variables were significantly correlated with mathematical achievement. Partial correlations were calculated simultaneously to examine the unique contribution that the attitude towards mathematics variable had with the logical thinking, line graphing in science and mathematical achievement variables. Following correlational analysis, statistically significant relationships between the predictor variables and the criterion variable were tested using hierarchical multiple regression. This method assessed the degree of variance in mathematical achievement as predicted by the attitudinal, line graphing, logical thinking measures, and was based on how the science and mathematics education literature defined the value of each variable when considering influences on mathematical achievement. Specifically, the attitude towards mathematics variable would be entered into the analysis as a forced entry first, followed by a block entry (employing stepwise analysis within the block) of the remaining predictors. Additional regression analysis was conducted in order to examine the amount of unique variance predicted by each individual variable for each regression model. Path analysis was conducted to determine the extent of direct, indirect and total effects among attitudes toward mathematics, line graphing and logical thinking on mathematics achievement. In this regard, the first set of variables were identified as exogenous; while the second, endogenous. This method was used to determine the extent of causality that the postulated variables had as part of the structural model. At the outset, testing of assumptions confirmed that
there was not an over presence of heteroscedasticity, non-linearity, or non-normally distributed residuals of the data used in the study. For the correlations, visual examination of the histograms, distribution curves and normal Q-Q plots indicated no apparent violations of assumptions of normality (Field, 2005).

Results

Correlational Analysis

Correlational analysis tested the null hypothesis that the relationship between mathematics attitude, line graphing, logical thinking and mathematics achievement would be equal to zero. This procedure was initially conducted to determine the suitability of predictors for the regression model, indicating statistically significant moderate-high correlations between mathematical achievement and line graphing ($r = .62, p < .001$) as well as logical thinking and mathematical achievement ($r = .60, p < .001$). Because of the moderate-low correlation found between mathematical achievement and mathematics attitude ($r = .46, p < .001$), partial correlations were then calculated for the logical thinking ($pr = .55, p < .001$) and line graphing ($pr = .55, p < .001$) variables and the mathematics achievement variable while controlling for mathematics attitude.

Analysis revealed that in addition to statistically significant correlations, mathematics attitude mediated 5-7% of the variability between the line graphing and logical thinking variables with mathematical achievement. It appears that attitude towards mathematics, when controlled, shares almost a tenth ($r^2 - pr^2 = .07$) of the portion of variance between line graphing and mathematical achievement. The slight reduction of variance between line graphing and mathematical achievement indicates that line graphing alone explains most of the variance, and that attitude plays a minor role in the relationship. Similarly, attitude towards mathematics, when controlled or held constant, shares a twentieth ($r^2 - pr^2 = .05$) of the portion of variance between logical thinking and mathematical achievement. The slight reduction of variance between logical thinking and mathematical achievement indicates that logical thinking alone explains most of the variance, and that attitude plays a minor role in the relationship. Although the individual contribution of attitude towards mathematics is small, it is nevertheless complex and cannot be ignored if one is to consider the role that attitudinal variables play in mathematical achievement.

Hierarchical Regression Analysis

Hierarchical regression analysis tested the null hypothesis that attitude towards mathematics, line graphing and logical thinking contributed nothing to the prediction of mathematical achievement. Embedded in the first hypothesis was the test that all regression coefficients for attitude towards mathematics, line graphing and logical thinking would be equal to zero when mathematical achievement was the criterion variable. Table 1 summarizes the results, indicating the amount of variance in mathematical achievement predicted by the attitude toward mathematics measure as well as the logical thinking and line graphing measures. Respective models A, B, and C show the amount of variance in mathematical achievement predicted by mathematics attitude alone, mathematics attitude coupled with line graphing, and mathematics attitude coupled with both line graphing and logical thinking. Although the predictors were entered in as a block during the analysis, the resulting stepwise calculation for these generated parsimonious models that explained a statistically significant amount of variance in each case. In model A, mathematics attitude was
found to predict a statistically significant amount of variance in mathematical achievement, as 21% of the variability was accounted for solely by mathematics attitude. Mathematics attitude was a statistically significant contributor to mathematics achievement ($F(1,85) = 22.30, p < .001$).

The impact that the mathematics attitude variable ($\beta =.46, \text{t}(85) = 4.72, p < .001$) had in model A indicated a moderate degree of contribution to the model. When line graphing was added in model B, the amount of variance in mathematical achievement explained by the two variables increased to 45%. The increase in predictability as a result of adding the line graphing variable was substantial, as both the mathematics attitude and the line graphing variables explain an additional 24% on the criterion variable. Line graphing and mathematics attitude were statistically significant contributors to mathematics achievement ($F(2,84) = 33.38, p <.001$). The impact that the line graphing variable ($\beta =.52, \text{t}(84) = 5.95, p < .001$) and mathematics attitude variable ($\beta =.27, \text{t}(84) = 3.09, p = .003$) had in model B were unbalanced in their respective contributions to the model, being that line graphing had a slightly greater impact than mathematics attitude. In model C, the mathematics attitude, line graphing and logical thinking variables account for a 9% increase in combined variability from the previous model. Overall, 54% of the variability on the mathematical achievement variable can be attributed to these three variables. Together, attitude towards mathematics, line graphing and logical thinking variables were statistically significant contributors to mathematics achievement ($F(3,83) = 31.77, p <.001$).

In models A, B and C the reported unstandardized coefficients appear to support the results, despite not being interpreted as a means to predict the mathematics achievement of any individual.

**Table 1. Hierarchical Regression Analysis Predicting Mathematical Achievement of Elementary Pre-service Teachers (N = 87)**

<table>
<thead>
<tr>
<th>Variable/Entry Order</th>
<th>B</th>
<th>SE B</th>
<th>$\beta$</th>
<th>R</th>
<th>R²</th>
<th>R²Δ</th>
<th>df</th>
<th>F-Ratio</th>
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<td>.21</td>
<td>1.85</td>
<td>22.30</td>
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<td>.07</td>
<td>.46***</td>
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<tr>
<td>Constant</td>
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<td>8.77</td>
<td>.45</td>
<td>.67</td>
<td>.24</td>
<td>2.84</td>
<td>33.38</td>
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<td>Mathematics attitude</td>
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<td>.06</td>
<td>.27**</td>
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<td>3.83</td>
<td>31.77</td>
<td>***</td>
</tr>
<tr>
<td>Mathematics attitude</td>
<td>.16</td>
<td>.06</td>
<td>.23**</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Line graphing</td>
<td>2.94</td>
<td>.74</td>
<td>.36***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Logical thinking</td>
<td>3.34</td>
<td>.83</td>
<td>.35***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note.* ** $p < .05$. *** $p < .001$. (1-tailed).
Path Analysis

Path Analysis tested the hypothesis that the regression weights for mathematics attitude (ATM1), line graphing (TOGS), and logical thinking (GALT) is equal to zero when mathematical achievement (MATH) is the criterion variable. An a priori structural model was constructed in AMOS 19 (IBM) according to the relationships discussed in the literature for the science and mathematics variables (see Figure 1). Examining this pre-determined relationship allows for a visual representation of the regression weights on the criterion variable as well as a depiction of implied causality with the arrangement. Figure 1 depicts the exogenous variables and endogenous variables, all of which are observed scores in the model. As such, exogenous and endogenous variables are depicted as rectangles. Inside each rectangle are the associated representations for each variable examined (i.e., ATM1, TOGS, GALT, MATH). Small circles with arrows pointing towards each rectangle represent measurement error terms (i.e., err1 – err3) associated with each observed variable. One-way, directional arrows represent the impact that one variable has on another.

![Figure 1. Theoretical path diagram for the influence of attitude towards mathematics, line graphing, and logical thinking.](image)

A Chi-square goodness-of-fit test supported by a number of goodness-of-fit indices and parameter estimates substantiated a structural model fit according to the correlation matrices of the sample data. Using multiple measures of model fit increases the likelihood of showing that hypothesized factor structures are maintained during either theory building or verification (Blunch, 2008). Specifically, the Chi-square goodness-of-fit test that achieves a low statistic with nonsignificance at the .05 level indicates a good fit. Hence, the calculated Chi-square statistic ($\chi^2$ [1, N = 87] = 1.64, p = .20) detected that the implied covariance and sample covariance was non-significant, indicating an acceptable fit with the data. Arguably, interpretation of this result should take into consideration the smaller sample size, whereby the statistic was regarded as insufficient in determining the extent of the fit of the model (Bryne, 2001).

In consideration of the risks associated with accepting a model based on small sample size, other goodness-of-fit indices were examined to further corroborate structural model fitting.
according to specifications discussed in the literature and include the following: (1) the Goodness of Fit Index, an absolute fit measure, (Jöreskog & Sörbom, 1984), with values approaching 1 indicating a good fit (as calculated: GFI = .99); (2) the Comparative Fit Index, a relative fit measure, (Bentler, 1990), with values approaching 1 indicating a good fit (as calculated: CFI = .99); and (3) the Root Mean Square Error Approximation, a fit measure based on non-central Chi-square distribution, (Brown & Cudeck, 1993), with values below .05 indicating a good fit and values no greater than .08 indicating realistic population errors of approximation (as calculated: RMSEA = .08). The RMSEA index approximates how well the model would fit the population covariant matrix based on unforeseen, but feasible, parameter values.

Analysis intended at identifying significant paths in the structural model indicated four out of five paths were statistically significant at the .001 level (ATM1 → TOGS, $SE_{b1} = .01, CR > 1.96$; TOGS → GALT, $SE_{b2} = .08, CR > 1.96$; GALT → MATH, $SE_{b3} = .81, CR > 1.96$; TOGS → MATH, $SE_{b4} = .73, CR > 1.96$), while the remaining path was statistically significant at the .01 level (ATM1 → MATH, $SE_{b5} = .06, p = .005, CR > 1.96$). All critical ratios greater than 1.96 were regarded as concurring evidence of statistical significance for each pathway (Bryne, 2001).

Standardized parameter estimates for the path model (see Figure 2) were all positive and are reported with their respective designations. Elementary pre-service teacher attitudes towards mathematics ($\beta = .23, p < .05$), line graphing ($\beta = .36, p < .01$), and logical thinking ($\beta = .36, p < .01$), were found to have a statistically significant direct effect on mathematics achievement. The squared multiple correlation estimates that the attitude towards mathematics predictor of line graphing explains 13% of its variance. Furthermore, the attitude towards mathematics and line graphing predictors both explain 26% of the variance on logical thinking. Overall, the attitude towards mathematics, line graphing and logical thinking predictors explain 53% of the variance on mathematical achievement. The total variability on the mathematical achievement outcome can be attributed to these three variables. These results indicate that pre-service teachers having more positive attitudes in mathematics, as well as increased aptitudes for science-based line graphing and logical thinking were more likely to have higher mathematics achievement.

Figure 2. Calculated path diagram for the influence of attitude towards mathematics, line graphing, and logical thinking.
When a variable has a mediating relationship with two other variables, it is considered to have an indirect relationship on the outcome variable (Preacher & Hayes, 2004). Thus, indirect effects add to unmediated effects calculated in a given path model (Fox, 1980; Kline, 1998). For example, the indirect effect of attitude towards mathematics on mathematics achievement is represented by a path from attitude towards mathematics to line graphing to mathematics achievement. Subsequently, line graphing is believed to mediate the effect of attitude towards mathematics on mathematics achievement. Indirect effects are calculated when the standardized regression coefficients from one variable path to another are multiplied in succession. As such, the indirect effect of attitude towards mathematics on mathematics achievement was found to be positive and statistically significant \(\beta = .19, p < .05\). This result suggests that the influence of pre-service teachers’ attitude towards mathematics is also mediated by line graphing in influencing mathematics achievement. Moreover, when line graphing and logical thinking serve as intermediary variables for attitude towards mathematics and mathematics achievement, the indirect effects are marginal, but nevertheless statistically significant \(\beta = .07, p < .05\). Hence, the total effect of attitudes towards mathematics on mathematics achievement is calculated as the summation of the direct and indirect effects, and is statistically significant \(\beta = .42, p < .05\). The indirect effect of line graphing on mathematics achievement was found to be statistically significant \(\beta = .18, p < .05\), suggesting that the influence of line graphing on mathematics achievement is also mediated by logical thinking. In this case, the total effect of line graphing in science was found to be statistically significant \(\beta = .54, p < .05\). Examining the standardized total effect parameter estimates for attitudes towards mathematics, line graphing and logical thinking, indicates that pre-service teachers’ performance in line graphing is the strongest predictor of mathematics achievement.

**Discussion and Implications**

The central purpose of this study was to examine the relationships among variables related to elementary pre-service teachers’ mathematical achievement. Included were elements of scientific cognition and attitudes towards mathematics. In ascertaining the predictive value of attitude, as well as science-based logical thinking and line graphing ability, this study employed a number of statistical procedures in order to determine the extent of influence that these variables had in the context of demonstrated mathematical achievement. Results were further substantiated with the inclusion of a path model configured according to what was depicted in the literature. Based on a Bandura’s (1986) theoretical model of reciprocal determinism, mathematical achievement was hypothesized to be a function of elementary pre-service teachers’ attitudes towards mathematics, line graphing and logical thinking abilities. Influential variables were hypothesized to have both direct and indirect effects, those of which were subject to examination through path analysis. In the main, both regression analysis and the operationalized causal model were found to be viable in corroborating statistically significant representations of relationships found herein.

Results indicate that the mathematical achievement of the elementary pre-service teachers sampled in this study was influenced, in part, by cognitive factors typically emphasized in science education. These results support the contention that knowledge and application of logical thinking and science-based line graphing were directly linked to the processes used in mathematics. Those who did not exhibit logical thinking or line graphing difficulties or possessed overly negative attitudes toward mathematics appeared to have increased mathematical abilities. This finding could have implications for the type of mathematics instruction provided at the in-service level. With the
exception of promoting positive attitudes towards mathematics, these factors are a function of the methods of science. Although the influence of cognitive factors might be accounted for with inquiry-based mathematics instruction, pre-service teachers being taught traditionally could perceive the incorporation of science-based line graphing and logical thinking as highly pragmatic or meaningful to their learning. If such factors were to be more conscientiously incorporated into mathematics teacher education, one could expect to see improved levels of mathematical knowledge. Lack of positive achievement outcomes has been shown to translate into circumstances where teachers with less content knowledge in mathematics incorporate less effective instructional practices (Fennema & Franke, 1992). In this regard, the use of pre-generated or pre-selected numbers, as well as coordinates for application while interpreting data could replace realistic, real-world data gathering. Undeniably, if some pre-service teachers go on to teach mathematics their level of mathematical knowledge may influence the extent to which they rely on more traditional formats to teach rather than more process oriented, inquiry-based methods (Rowland, Huckstep, & Thwaites, 2005).

Bandura’s (1986) SCT would suggest that the logical thinking, line graphing, and attitude predictors found in this study did satisfy the cognitive dimension of the reciprocal determinism model, leaving the remaining portion of variance to be explained by additional cognitive, or other environmental and behavioral factors. The main postulate of Bandura’s SCT would specify that the mathematical achievement outcome would be comprehensively influenced by these three dimensions. The results of this study are associated with one of these three dimensions, being that 54% of the variance was accounted for in the final regression model, capturing the presence of two statistically significant cognitive factors and one statistically significant attitudinal factor. This result was further verified as 53% of the variance for these factors was accounted for during path analysis. Therefore, SCT would concur that the three factors examined in this study did play an important role in explaining mathematical achievement.

Previous research related to mathematical achievement and line graphing found that mathematical abilities accounted for 45-48% of the variability on a graph comprehension outcome when used as a predictor, and that correlations near .70 were found between these variables (Curcio, 1987). The results of the current study resemble these findings, especially when the predictor variable is line graphing and the criterion variable is mathematical achievement. Additionally, research investigating the relationship between logical thinking and line graphing found that logical thinking is necessary for individuals to successfully construct and interpret line graphs (Berg & Phillips, 1994). The results of the current study support the idea that these variables together have important ramifications for the mathematics achievement of the given sample of elementary pre-service teachers.

Courses in mathematics methods have the potential to improve pre-service elementary teacher performance in mathematics by emphasizing logical thinking and line graphing. Achieving this may require the instructor to be more cognizant of how some elements of science could serve to improve the mathematical abilities of pre-service teachers being taught during methods instruction. Teacher education programs must not only focus on the improvement of content knowledge of pre-service teachers, but also assist in developing more positive attitudes towards mathematics. As future teachers, these individuals will be using a number of different strategies during instruction, some of which could incorporate mathematics and science integration. Extending training to include the factors examined in this study could shape teacher training program philosophies to seriously consider the influence that attitudes have on content knowledge and related forms of cognition. Future research should seek to explain a greater portion of the variability on
mathematical ability; possible predictors for further study might include quality of mathematical instruction, environmental conditions, as well as affective variables such as mathematics anxiety or self-efficacy.

Limitations

The approaches used to examine the factors in this study were used to establish an understanding of the influences that each predictor variable had on the outcome variable of mathematics achievement for a group of elementary pre-service teachers. Although not considered in this study, similar statistical approaches could be used to forecast individual performance outcomes given different interpretations of the same regression analysis. In that regard, certain statistics (i.e., B, SE B) reported were not only used to assist in determining the unique contribution of each variable within the respective regression model, but were included for those interested in estimating how each factor responded to specific units of change associated with a paired variable or set of variables.

The arrangement of the path analysis variables used to corroborate the regression results may present limitations insofar that causality may not be sufficiently established – albeit implied theoretically. It is altogether possible that other variables not included as part of this study could further account for the relationships investigated. Moreover, that mathematical achievement could be causally linked to other behavioral or environmental determinants as presented in SCT (Bandura, 1986). Perhaps the mathematical achievement of pre-service teachers is influenced by reading comprehension skills, content specific literacy (Österholm, 2006a, 2006b) or semiotic representations of information unique to experience level, understanding and background (Roth, 2002).

The statistical configuration of the path model that was tested in this study was based on relationships derived from the literature. Therefore, other possibilities of interrelating variables not examined as part of that review may offer additional insight or account for a greater portion of variability in both the regression and path way models. Because the participants in this study were sampled from a population of pre-service elementary education teachers from a research-based university in the Midwest, it is important that the study be replicated in other populations with similar contexts. One should consider the nature and context of the teacher education program, other factors not examined as part of this study, as well as combinations of other factors in accordance to the remaining two-thirds of Bandura’s model of reciprocal determinism. Despite these limitations as well as those associated with generalizability, this study offers some important insights regarding the relationships among pre-service teachers’ mathematical achievement, science-based cognition and attitude.

References


